Note on Lorentz Contractions and the Space Geometry of the Rotating Disc

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An apparent paradox concerning the space geometry of the relativistic rotating disc is resolved.

The space geometry of a rotating disc is discussed in many papers and textbooks [1]. It is generally agreed that the relativistic Lorentz contraction shortens the circumference of the disc by a factor $\sqrt{1-\omega^2 r^2/c^2}$, whereas the radius retains its rest length r. Thereby the space geometry on the rotating disc becomes non-Euclidean.

The purpose of this note is mainly pedagogical. An apparent paradox will be described which arises in this connection. From the resolution of this paradox one may learn two things. First, one realizes that there are, in a certain sense, *two different kinds of Lorentz contractions*. Secondly, one sees most clearly *how the non-Euclidean space geometry comes into being* if the disc is set rotating.

Consider the following arrangement. Measuring rods of length l are placed along the x axis of an inertial system, with empty distances of length l between them. Now look at this arrangement from another inertial system, which moves with constant velocity v in the x direction. Then, due to the Lorentz contraction, the *rods* as well as the *distances* between them appear shortened by the *same* factor $\sqrt{1-v^2/c^2}$.

Now consider the following model of a rotating disc. N equal rods R of length r are fixed with one endpoint to a common rotation axis, so that they can rotate independently in a plane orthogonal to this axis. Assume first these rods to be at rest in some inertial system, with equal angles $2\pi/N$ between neighboring rods. To the free endpoints of these radial rods, N other equal rods L of length $l=\pi r/N$ are rigidly attached along the circumference of a "wheel". For $N \gg 1$, the empty distances between two neighboring rods L are also aproximately equal to $l=\pi r/N$.

Assume this wheel to be set rotating, e. g., be means of N equal rockets which are attached along the rods L and which are fired simultaneously. Some time after these rockets have stopped burning, all elastic oscillations of the system which necessarily have been excited by the time-dependent driving forces will die away, and the wheel will keep rotating with constant angular velocity ω. Let the rods be strong enough to withstand without deformations the time-independent centrifugal forces caused by this rotation. How, then, does the rotating wheel look like?

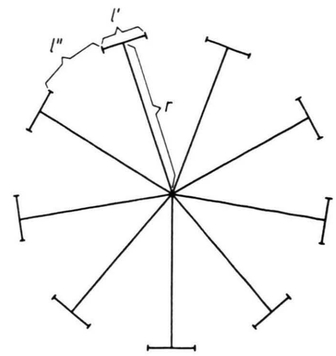

Fig. 1.

Consider the projection of the rotating wheel into the space coordinates of the original inertial system at some fixed time t_0 (Fig. 1). For symmetry reasons, the angle between any two neighboring radial rods R on this projection is $2\pi/N$, as for the wheel at rest. However, whereas the radial rods have retained their length r, the tangential rods L now are *contracted*, their new length being

$$l' = l\sqrt{1-\omega^2 r^2/c^2}\,. \tag{1}$$

They thus occupy a total length $N l' < \pi r$, whereas the whole circumference of the wheel is $2\pi r$. One is thus forced to conclude that the empty distance between two neighboring tangential rods has *expanded* from its original value l to

$$l'' = l(2-\sqrt{1-\omega^2 r^2/c^2}), \tag{2}$$

in order that $N l' + N l'' = 2\pi r$. The rods L and the distances between them thus behave *differently* if the wheel is set rotating.

The example considered before, however, suggests that *both* rods and "empty space" will be *shortened* under Lorentz contractions by the same factor $\sqrt{1-v^2/c^2}$. Therefore, it seems to be wrong to explain the non-Euclidean geometry on a rotating disc simply with the help of Lorentz contractions. One might then be tempted to look for a different explanation, or even to doubt the whole effect altogether.

This aparent paradox may, however, be resolved easily by means of another thought experiment. Consider again the arrangement of rods in the first example. But now, instead of looking at it from a moving inertial system, the whole arrangement will be set moving with uniform velocity v along the x axis. This may again be performed by means of equal rockets, attached to the rods and fired simultaneously at time $t=0$. Fig. 2 shows the original arrangement, the worldlines of the left endpoints of the rods, and a picture of the moving rods at time $t=t_0$. Since the spatial distance between two neighboring worldlines is always $2l$ whereas the moving rods got *contracted* from l to

$$l' = l\sqrt{1-v^2/c^2}\,, \tag{3}$$

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[1] References may be found in the recent paper by G. CAVALLERI, Nuovo Cim. **53 B**, 415 [1968]. Compare also V. CANTONI, Nuovo Cim. **57 B**, 220 [1968].

the distances between the rods must have *expanded* from l to

$$l'' = l(2 - \sqrt{1 - v^2/c^2}). \tag{4}$$

This corresponds exactly to Eqs. (1) and (2) above.

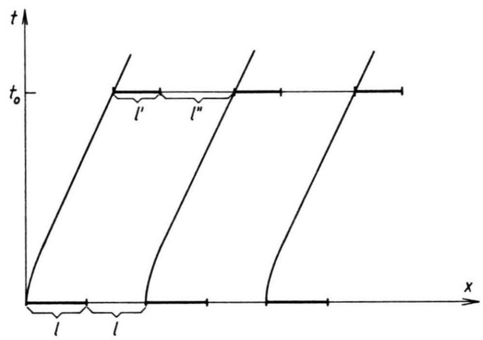

Fig. 2.

Comparing the first and third example, one arrives at the following conclusion. There are two kinds of Lorentz contractions. A rod, being at rest in an inertial system, *appears* contracted if seen by a moving observer. This kind of Lorentz contraction does not imply any real change of the rod, but merely accounts for the different "space-time perspectives" of observers in relative motion. If, however, the rod itself is set moving, one is justified to say that *it gets really contracted.*

Indeed, in the latter case the rod contracts "with respect to the surrounding space", as indicated, e. g., by Eqs. (3) and (4). That this contraction is a real physical process, may also be illustrated by a slight modification of the experimental arrangement of Fig. 2. Imagine neighboring rods to be connected by springs of equilibrium length l, and regard the whole chain of rods together as a model of a single rod. Let the periods of all possible oscillations of the chain be much greater than t_0. Then Fig. 2 remains nearly correct, but the drawn configuration at $t = t_0$ is not yet stable, since the springs are stretched and the chain just starts oscillating. After these oscillations have died away by damping,, the chain as a whole has been shortened by the famous factor $\sqrt{1 - v^2/c^2}$.

Although this factor occurs in both types of Lorentz contractions of a rod (and this is not an accident, but simply expresses the invariance of its restlength), they are rather different things.

As clearly indicated by Eqs. (1) and (2), the non-Euclidean geometry on a rotating disc is due to Lorentz contractions of the "real" type. This remark and the above described model should be sufficient to explain why and how the space geometry changes if a disc is set rotating.

Isotope Effect of Thermal Diffusion in Some Pure Molten Salts

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The heat of transport for isotopes of the cations has been determined for the following molten salts: KCl (78 cal/mole), KBr (46 cal/mole), KSCN (52 cal/mole), RbCl (92 cal/mole), and RbBr (49 cal/mole). Thus, the heat of transport is roughly the same for corresponding potassium and rubidium salts, and it is considerably higher for a chloride than for the other ones. In these two aspects the isotope effect of thermal diffusion behaves differently from that of electromigration in molten salts.

Thermal diffusion in solid and molten salts has been investigated in this laboratory for a number of years. One of the main objects has been to study the separation of isotopes of alkali metal ions, for which we have previously given results for sulfates and nitrates [1–4], and for which we now can report on same halides and a thiocyanate.

As in the previous studies a temperature difference was maintained between the bottom and top of narrow glass tubes containing the melt, cf. Table 1. After quenching, the cell was divided into samples for which the isotope abundance ratio was measured. Since thermal diffusion causes only a small change in the isotope abundance, we have always been faced with the problem of how to get significant results without being forced to do a great number of experiments. We have sometimes chosen to divide the whole column into samples, analyse each of them, and then use regression analysis in order to calculate the temperature coefficient of the isotope abundance ratio. Other times we restricted the analysis to samples from the top and the bottom of the cell. An advantage of the first method is that it might be possible to detect a temperature dependence of the thermal diffusion; a disadvantage is that uncontrolled mixing might occur in the tube during solidification, which would cause the measured temperature coefficient to be too low [5]. A comparison

[1] K. Lindqvist and A. Lundén, Z. Naturforsch. **16 a**, 626 [1961].
[2] S. Gustafsson and A. Lundén, Z. Naturforsch. **17 a**, 550 [1962].
[3] S. Gustafsson, Z. Naturforsch. **18 a**, 949 [1963]; **21 a**, 842 [1966].
[4] A. Lundén, Z. Naturforsch. **24 a**, 1673 [1969].
[5] cf. V. Backlund, J. Dupuy, S. Gustafsson, and A. Lundén, Z. Naturforsch. **22 a**, 471 [1967].